XV. *Of the Temperament of those musical Instruments, in which the Tones, Keys, or Frets, are fixed, as in the Harpsichord, Organ, Guitar,* &c. *By Mr.* Tiberius Cavallo, *F. R. S.*

Read April 3, 1788.

THE scale of music, which is used at present, consists of seven principal notes or sounds, which musicians denote by the letters of the alphabet A, B, C, D, E, F, and G; which, together with some intermediate ones, commonly called flats and sharps, and the octave of the first, make 13 sounds.

When those sounds are considered with respect to the first, they are called by the following names, *viz.* the prime or key-note, the second minor, second, third minor, third major, fourth, fourth major, fifth, sixth minor, sixth major, seventh minor, seventh major, and octave.

Musical sounds are produced by the vibrations of the sonorous bodies, and they are acuter or graver as the vibrations performed in a given time are more or less in number; so that if a string vibrating 100 times in a second produces a certain sound, and another string vibrating 120 times in a second produces another sound, the latter is said to be acuter, higher, or sharper than the former.

The number of vibrations performed in a certain time principally depends on the thickness, length, and elasticity of the sonorous bodies; but as the simplest sonorous bodies, and the fittest for examination, are those strings which are equal in every

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other

other reſpect, excepting in their lengths, becauſe the number of vibrations, which they perform in a given time, is ſimply in the proportion of their lengths, we ſhall conſider only thoſe in the preſent inveſtigation, the number of vibrations per-formed by other ſorts of ſonorous bodies being eaſily deduced from them.

As the above-mentioned 13 ſounds are all different from each other, the ſtrings which produce them differ in length, and of courſe in the number of the vibrations, which, when ſtruck, they perform in a certain time. Here follow the pro-portions which the numbers of vibrations performed in a given time, or the length of the ſtrings which expreſs thoſe 13 ſounds, bear to the firſt, prime, or key-note.

Firſt	1	Fourth	$\frac{3}{4}$	Seventh minor	$\frac{5}{9}$
Second minor	$\frac{15}{16}$	Fourth major	$\frac{32}{45}$	Seventh major	$\frac{8}{15}$
Second	$\frac{8}{9}$	Fifth	$\frac{2}{3}$	Octave	$\frac{1}{2}$
Third minor	$\frac{5}{6}$	Sixth minor	$\frac{5}{8}$		
Third major	$\frac{4}{5}$	Sixth major	$\frac{3}{5}$		

If, inſtead of many ſtrings having thoſe lengths in order to expreſs the 13 ſounds, or notes of an octave, one ſtring be divided according to thoſe proportions, and this ſtring be ſtopped conſecutively in the different points or diviſions; on being ſtruck, it will expreſs the correſponding ſounds. Thus, if a ſtring ſtretched between two fixed points, as CZ fig. 1. Tab. III. be ſtruck, it will produce a ſound called the prime, firſt or key-note ; if it be ſtopped in the middle, one-half of the ſtring will ſound the octave, its length, compared to that of the whole ſtring, being in the proportion of 2 to 1 ; if two-thirds of the ſtring be cauſed to vibrate, the ſound produced will be the fifth, its

length,

length, compared to that of the whole ſtring, being as 2 to 3, and ſo of the reſt.

The higheſt ſound of the octave is expreſſed by the half of the ſtring ; and if this half be divided again in the ſame manner or proportion, a higher octave will be obtained, the higheſt note of which will be expreſſed by a quarter of the original ſtring. This quarter may be divided again into a higher octave, and ſo on ; therefore, a ſtring ſo divided may expreſs the ſounds of all the keys of an harpſichord or organ.

In regard to thoſe diviſions it muſt be obſerved, that as the notes of the ſecond octave bear the ſame proportion to the firſt note of that octave as the notes of the firſt octave reſpectively bear to the firſt note of that octave, or to the whole ſtring; and as the length of the ſtring expreſſing the firſt note of the ſecond octave, *viz.* cZ, fig. 1. is half the length of CZ, the firſt note of the firſt octave, it follows, that the length of the ſtring of every note in the ſecond octave is half the length of the correſponding note in the firſt octave ; thus g in the ſecond octave is 120 inches long, and G in the firſt octave is 240 inches long, *viz.* twice 120. Hence, when the diviſions of the firſt octave are aſcertained, in order to find the diviſions of the notes of the ſecond octave, we need only take the half of the lengths expreſſing the notes in the firſt octave. By the very ſame reaſoning it is evident, that to find the diviſions for the third octave, we need only take the halves of the lengths which expreſs the notes of the ſecond octave, or the quarters of thoſe of the firſt octave, and ſo of the reſt.

The ſtring or line CZ, fig. 1. is divided in the above-mentioned manner, and in order to avoid confuſion, the diviſions of the principal notes only of the firſt and ſecond octave are annexed to it. The numbers under the line expreſs the lengths

from

from Z to the divifions to which they ftand near. The letters
juft over the lines are the names of the notes or founds ex-
preffed by the correfponding lengths of the ftring. The frac-
tional numbers exprefs the proportion which each particular
divifion bears to the whole ftring; and the Roman numbers
denote the numerical names of each note with refpect to its
diftance from the firft, which is always included. For example,
fuppofe the whole ftring to be called C, and to be 360
inches long: then, if this ftring be ftopped in G, the part GZ
will be 240 inches, *viz.* two-thirds of the whole ftring CZ;
the found expreffed by it when ftruck is called G, and it is
the fifth note from C, which is the firft or key-note. Again,
if this ftring be ftopped in A, the part AZ will be equal to
216 inches, *viz.* three-fifths of the whole; the found pro-
duced by it is called A, and is a fixth to the key-note C, &c.

It is evident, that if any of thofe divifions be confidered as
the firft or key-note, then the other notes, though they retain
their alphabetical names, muft have their numerical names
altered accordingly: for example, if we take D for the key-
note, then A will be the fifth of it, whereas A was the fixth
when C was confidered as the key-note; thus alfo B is the
third of G, and the feventh of C, and fo on.

Thus much having been premifed, we may proceed to fhew
the meaning of what is called *the temperament* in a fyftem of
mufical founds, and likewife the neceffity of it. For this pur-
pofe it is neceffary to recollect, firft, that the ftring, divided in
the above-mentioned manner, exhibits the various notes or
founds of the keys of an harpfichord, the pipes of an organ,
&c. Secondly, that thofe divifions remain unalterable, fo that
the harpfichord, when tuned, cannot be altered in the courfe
of performing on it; and, thirdly, that when any one of
thofe

thoſe notes or diviſions is conſidered as the key-note, its ſecond, third, fourth, fifth, &c. muſt bear their reſpective proportions, according to what has been ſaid in the preceding pages.

Now, if amongſt the diviſions of the ſtring CZ, fig. 1. we take D for the firſt or key-note, its length being 320 inches, the length of its fifth muſt be 213⅓ inches, *viz.* two-thirds of 320, that being the proportion which the fifth muſt bear to the key-note; but amongſt the diviſions of the ſtring, there is none equal to 213⅓ inches; therefore, there is not a note among them which may ſerve for a fifth to D: however, as the length of AZ, *viz.* 216, is the neareſt to 213⅓, this A muſt be taken for the fifth of D. It is evident, that this is an imperfect fifth of D; but if, in order to render it perfect, we make AZ equal to 213⅓ inches inſtead of 216, then it will be a redundant ſixth to C, when C is conſidered as the key-note; the beſt expedient, therefore, is to divide the imperfection between the two lengths, *viz.* to make AZ neither ſo long as 216, nor ſo ſhort as 213⅓, which will render the diſagreeable ſenſation, ariſing from the improper length, the leaſt poſſible. This alteration of the juſt lengths of ſtrings, neceſſary for adapting them to ſeveral key-notes, is called *the temperament :* and the beſt temperament in a ſet of muſical ſounds is evidently ſuch a partition of the natural imperfections, as will render all the chords equally and the leaſt diſagreeable poſſible.

What has been exemplified in D and A may be ſaid of all the other notes; ſo that if any one of them be a perfect third, fifth, &c. with reſpect to one key-note, it will be found to be imperfect with reſpect to others. Hence it is manifeſt, firſt, that in a ſet of muſical keys, pipes, or frets, a temperament is abſolutely neceſſary; and, ſecondly, that the harpſichord,

organ,

organ, guitar, or any other inftrument in which the notes are fixed, fo as not to be alterable by the performer's hands, muft be imperfect even when tuned in the beft manner poffible; for by the temperament we can divide, but not annihilate, the imperfection.

Other inftruments, in which the notes are not fixed, as the violin, violoncello, &c. are perfect, becaufe the performer ftops the ftrings upon them in different places, even for founding the notes of the fame name. Thus a fkilful performer, in order to found A, will ftop the ftring a little farther from the bridge when he plays in the key of C, *viz.* when C is confidered as the key-note, than when he plays in the key of D.

Moft people imagine, that the fcale of mufic is capable of many different temperaments; and, agreeable to this fuppofition, the writers on harmonics have propofed different temperaments; but it will be fhewn in the fequel, that the nature of the fcale admits of only one temperament capable of rendering the imperfection and the harmony equal throughout; and that it is impoffible to form a different and more advantageous fcale.

Before we begin with the inveftigation of this fubject, it will be neceffary to explain certain principles, the want of which may poffibly raife fome doubts in the minds of thofe perfons who are not much acquainted with the theory of mufical founds. In the firft place it muft be obferved, that the proportion of 2 to 3 for the fifth, the proportion of 1 to 2 for the octave, and in fhort the proportions of all the notes, are not affumed at pleafure; but they have been determined from conftant experience, *viz.* from the agreeable or difagreeable effects produced when two different notes are founded at the fame time.

To render this more evident, let two ſtrings equal in every
reſpect be ſtruck at the ſame time, and they will exprefs the
ſame found precifely, ſo that no ear can perceive any dif-
ference between them, and it is almoſt impoſſible to diſtin-
guiſh whether the ſound ariſes from two ſtrings, or from one
only, excepting from the loudnefs. But if one of thoſe
ſtrings be fucceſſively ſtopped in different parts of its length,
whilſt the other remains open as before, and if at every time
they be both ſtruck together, their combined founds will be
found to produce different effects, *viz.* ſometimes more or lefs
pleafing, and at other times more or lefs difagreeable. When
the combinations of the two founds are agreeable, they are
called *concords*; and when difagreeable, they are called *difcords.*

Experience evinces, that the beſt concord is when the length
of one ſtring is to the length of the other as 1 to 2, every
other circumſtance being the ſame in both. This proportion
forms the octave. The next beſt concord is the fifth, *viz.*
when the lengths of the two ſtrings are as 2 to 3, after which
come the proportions of 3 to 4, 4 to 5, 3 to 5, 5 to 6, and
5 to 8, for the other concords. The other proportions befides
thefe are difagreeable in a greater or lefs degree, unlefs they
are greater than the proportion of 1 to 2; but in that cafe it
will be found, that the proportions which produce agreeable
combinations are the double, quadruple, octuple, &c. of thoſe
mentioned above, *viz.* are their octaves, double octaves, &c. :
thus the proportion of 1 to 4 produces a very agreeable con-
cord, becaufe 1 to 4 is the double of 1 to 2, *viz.* it expreſſes a
double octave.

Secondly, it appears, from the foregoing obfervations, that
if we have the length of a ſtring, or the proportion of a note
in any part of the ſtring, we may eafily find its octaves by
                                                        taking

taking its double, or its half, or the double of the double, &c.: for inftance, in fig. 1. if cZ be given equal to 90 inches, we may find its octave below by taking twice 90, *viz.* 180, or the octave of this octave, which is 360, *viz.* equal to twice 180, or to four times 90; and, on the other fide, we may find the octave above of the given note by taking its half, which is 45, &c.

It is now neceffary to fhew why within the octave there are admitted only thirteen different notes, *viz.* eight principal ones, and five others, called fharps and flats.

The line XY, fig. 2. reprefents a mufical ftring, the length of which is fuppofed to be divided into 13286025 equal parts *. On one fide of this line there are the divifions of feven fucceffive octaves, *viz.* the half of XZ, a quarter of it, &c.; and on the other fide are the divifions of a feries of fifths, *viz.* the fifth of the whole ftring, the fifth of this fifth, and fo on, which are found by taking two-thirds of the whole ftring, then two-thirds of thofe two thirds, and fo on.

Here we take notice only of the octaves and fifths, becaufe they are the principal and the beft concords; fo that a temperament being required, it is neceffary firft to take care, that thefe concords be not rendered infufferable to the ear, the reft admitting of a greater latitude in the temperament or deviation from the perfect ftate. Befides, it will appear in the fequel, that all the other notes are derived from the feries of fucceffive fifths.

In whatever key a piece of mufic is performed, its fifth is the moft predominant of its concords; and as the notes of

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* This number has been chofen, becaufe the line may in that cafe be divided into the neceffary number of fucceffive octaves and fifths, without any common fractions, which renders the operation more eafy and more perfpicuous; otherwife there might have been affumed any other number.

mufic

mufic muſt be ſo ordered as that, for the ſake of modulation, any note may be conſidered as the key-note; therefore having found the fifth of the whole ſtring by taking two-thirds of its length, which gives a note called G, we muſt ſuppoſe, that this G may be conſidered as the key-note, conſequently muſt find its fifth, which gives D, as ſhewn in the figure, and ſo on, until we find one of thoſe ſucceſſive fifths, which coincides with one of the ſucceſſive octaves; for after that, to find more ſucceſſive fifths would be only repeating the ſame thing over again.

Indeed, if we carry the ſucceſſion of octaves and of fifths indefinitely far, we ſhall find, that no one of the fifths ever coincides perfectly with one of the octaves, and therefore the diviſion would have no end. However, as the length of the ſeventh octave comes ſo very near to the twelfth fifth, we muſt be contented with taking this ſeventh octave for the fifth of F, the difference between this note and the perfect fifth of F being about the hundredth part of its length; whereas, if we carry on the ſucceſſion of fifths and of octaves, we ſhall find, that amongſt thirty and more fifths none comes nearer to one of the octaves than the above-mentioned one, as may be ſeen in the following table, which contains a ſeries of ſucceſſive octaves, and another ſeries of ſucceſſive fifths, as in fig. 2.; but in the table the diviſions are carried farther on than in the figure:

Octaves.	Fifths.
13286025	13286025.
6643012,5	8857350.
3321506,25	5904900.
1660753,125	3936600.
830376,5625	2624400.

Octaves

Octaves.	Fifths.
415188,28125	1749600.
207594,140625	1166400.
☞103797,0703125	777600.
51898,5351 +	518400.
25949,2675 +	345600.
12974,6387 +	230400.
6487,3193 +	153600.
3243,654 +	☞102400.
1621,827 +	68266,6 +
810,913 +	45511 +
405,456 +	30340,7 +
202,728 +	20227 +
101,364 +	6742,3 +
50,662 +	4494,8 +
25,331 +	2996,6 +
12,665 +	1997,7 +
	1331,8 +
	887,8 +
	592 +
	394,6 +
	263 +
	175,4 +
	117 +
	78 +
	52 +
	34,6 +
	23 +
	15,3 +

The number of fifths then in this feries is twelve, and as, from what has been faid above, when the divifion expreffing a certain note has been affigned in any part of a ftring, we may

2   eafily

eafily find all its octaves above and below, it follows, that by finding all the octaves of thofe twelve divifions we fhall have twelve diftinct notes within half the ftring, *viz.* within the firft octave of the whole ftring; to which, if the found of the whole ftring be added, we fhall have thirteen different founds, which fhews why an octave comprehends neither more nor lefs than thirteen notes.

Without dwelling any longer upon the names or number of thofe notes, I fhall immediately proceed to find out the temperament.

It has been fhewn above, and it is expreffed in fig. 2. that the length of the ftring for the laft fifth is fhorter than the length of the laft octave, and alfo that one of them muft be neceffarily taken for both purpofes; but here we muft confult nature, examining by the ear which of the two is leaft difagreeable. This, however, is foon decided; for imperfect octaves are quite infufferable, whereas a certain degree of imperfection in the fifths is tolerable; therefore we are neceffitated to leave the octaves perfect, and to let the feventh octave ferve for the fifth of F. In this cafe it is evident, that each of the notes in the fucceffion of fifths is a perfect fifth to its preceding note, excepting the laft, which would be by much too flat, and therefore it is neceffary to divide the imperfection equally among them all.

For this purpofe it muft be confidered, that as the twelve fucceffive fifths, together with the whole ftring or firft note, are each two-thirds of its preceding note; they form a geometrical feries, the ratio of which is $\frac{2}{3}$, its extremes are 13286025 and 102400, and the number of terms is 13. But becaufe inftead of 102400, which is the laft fifth, we muft take the number 103797,0703125 (*viz.* the length of the laft octave)

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for the laft term of the feries; therefore the problem is reduced to the finding out of eleven mean proportionals between the two numbers 13286025 and 103797,0703125.

It is demonftrated in almoft every treatife on algebra and arithmetic, that in a geometrical progreffion, as the above-mentioned one, the firft or fmalleft extreme is to the laft or greateft extreme as unity is to a power of the ratio, the index of which is equal to the number of terms lefs one. Hence, in our cafe, in which the number of terms, including the two extremes, is 13, we fhall have 103797,0703125 : 13286025 :: 1 : R^{12}, from which the ratio is found by dividing the fecond number by the firft, and extracting the twelfth root from the quotient, *viz.* $\dfrac{13286025}{103797,0703125.} = 128$; and $\overline{128}|^{\frac{1}{12}} = 1,4983069$, which is the ratio fought.

The ratio having been afcertained, the fucceffion of tempered fifths is thus eafily determined; *viz.* divide the length of the whole ftring by this ratio, and the quotient gives the firft tempered fifth; divide this fifth by the fame ratio, and the quotient gives the fecond tempered fifth; divide this fecond fifth by the fame ratio, and fo on till the laft fifth, which comes out equal to 103797, 21735 (fee fig. 3.) which is fo nearly equal to the length of the feventh octave, that the difference is truly infignificant; but, if greater accuracy were required, we need only extract the proper root of 128 to a greater number of decimals.

Fig. 3. fhews the divifions of the ftring XZ tempered in the above-mentioned manner; *viz.* the fucceffive fifths have been afcertained firft, and then, by taking their octaves, the whole fet of divifions has been completed. By comparing this figure with fig. 2. one may eafily perceive, how fmall is

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the difference between the perfect fifths of the latter, and the
tempered ones of the former.

The divifions, thus afcertained, form a feries of notes, in
which the octaves only are perfect; but all the fifths, all the
thirds, and in fhort all the chords of the fame denomination,
are equally tempered throughout: fo that whichever of them
is taken for the key-note, its fifth, fixth, &c. will have al-
ways the fame proportion to it, and confequently will always
produce the fame harmony when founded with it.

It is evident, that, befides this, there can be no other tem-
perament capable of producing equal harmony; for when the
extremes of a geometrical feries and number of mean propor-
tionals are given, there can be but one fet of thofe means:
thus, if we are to find two mean proportionals between the
numbers 2 and 16, thefe are neceffarily 4 and 8; nor is it
poffible to affign any others.

If, on the other hand, we endeavour to find a better tem-
perament by introducing more than thirteen notes within the
limits of an octave, we fhall find it impracticable, becaufe it
has been fhewn, in the preceding pages, that after the number
thirteen, if the fucceffion of fifths be carried farther on, they
will recede more from a coincidence with any one of the
octaves.

This explanation of the nature, origin, and neceffity of the
temperament has been thought neceffary for the fake of per-
fpicuity; but the fame end may be obtained by the following
eafier method. As the thirteen notes of an octave muft be
arranged fo, that whichever of them be taken for the firft or
key-note, the fecond, third, fourth, &c. may bear the fame
conftant proportion to it; therefore it follows, that they muft
be in a geometrical proportion one of the other, fo as to form a

ſeries of thirteen numbers, the extremes of which are the whole ſtring and its half, *viz.* any number and its half. The ratio of this ſeries is found in the ſame manner as in the other ſeries, *viz.* the greateſt extreme is divided by the leaſt, and the twelfth root of the quotient is the ratio ſought. But the extremes are any aſſumed number and its half: and as the quotient of a number divided by the half of the ſame number is always equal to two; therefore, whatever be the length of the ſtring, the ratio is always $\overline{2}^{\frac{1}{12}} = 1,0594+$, and if the length of the whole ſtring be divided by this ratio, *viz.* 1,0594+, the quotient will be the length of the ſtring expreſſing the ſecond note, which, divided by the ſame ratio, gives the third note, and ſo on; or elſe, inſtead of dividing the length of the whole ſtring by the ratio, you may multiply the half of it by the ratio, the product of which will give the ſeventh note, which multiplied by the ſame ratio gives the ſixth, and ſo on in a retrograde order, which will give the tempered notes of the octaves as well as the former method. By this means the following diviſions for the notes of an octave have been calculated, the length of the whole ſtring having been ſuppoſed equal to 100000.

I.   100000.
✻ ♭   94387.
II.   89090.
✻ ♭   84090.
III.   79370.
IV.   74915.
✻ ♭   70710.
V.   66743.
✻ ♭   62997.
VI.   59462.
✻ ♭   56123.

VII.   52973.

VIII.   50000.

If a monochord be divided in this manner, and a harpſi-chord tuned by it, this inſtrument will then be tuned ſo, that whichever note be taken for the firſt or key-note, its fifth, ſixth, &c. will produce the ſame effect reſpectively.

Thus far I have endeavoured to explain this ſubject in the moſt familiar manner, avoiding as much as poſſible the mathe-matical language and ſymbols; having found, by experience, that intricate mathematical diſquiſitions, eſpecially on this ſubject, are underſtood only by a few able mathematicians, but that they are neither comprehended, nor even read, by thoſe who might wiſh to underſtand, or to uſe them. It is now neceſſary to conſider this ſubject with reſpect to the practice.

At preſent, the harpſichords and organs are commonly tuned ſo, that ſome concords are very agreeable to the ear, whilſt others are quite intolerable; or, in other words, when the performer plays in certain keys, the harmony is very plea-ſing, in others the harmony is juſt tolerable, and in ſome other keys the harmony is quite diſagreeable.

The beſt keys to be played in are the keys of C, of F, of E flat, of B flat, of G and of D in the major mood, and the keys of C, of D, of A, and of B, in the minor mood. Next to thoſe come the leſs agreeable keys of A, A flat, and E in the major mood; beſides thoſe, the reſt are diſagreeable in a greater or leſs degree, ſo that out of twelve keys *, which, on account of the two moods, *viz.* the major and the minor, become twenty-four, there are hardly fourteen that can be

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* The octave muſt be excepted, in this reſpect it being the ſame as the firſt note.

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used; and for this reason most of the modern compositions in music are written in those keys.

So far the common method of tuning answers some purpose; for as long as the performer is to play in certain keys only, it is much better to have them tuned in the most advantageous manner, than to let those be tuned in a less perfect manner for the sake of others, which he does not intend to use. Hence the great harpsichord players generally have their instruments tuned in a peculiar manner, *viz.* so as to give the most advantageous effect to those concords which they more frequently use in their compositions. And hence also, the harpsichords and organs are always tuned different from each other, unless they be tuned by the same person with equal attention, and without any particular instructions.

This practice cannot conveniently be laid aside, *viz.* when the instrument is to be tuned for solo playing; and for a certain style of music, it is very proper to tune it so as to give the greatest effect to those combinations of sounds, which are mostly used in those compositions. But the case is far different when the instrument is to serve for accompanying other instruments in every sort of music, or the voices of good singers; for then the disagreement becomes very audible; and for this purpose the harpsichord or organ ought to be tuned according to the above demonstrated temperament of equal harmony, which is the only one that can possibly take place.

When the compositions of old masters are performed in concert, and with the organ or harpsichord tuned in the common manner, the effect is frequently very disagreeable. This is particularly the case with the songs of HANDEL, GALLUPPI, LEO, PERGOLESE, and others, who wrote in a great variety

of

of keys, and very often in thoſe, for which the common way of tuning is not at all calculated.

In order to hear the effect of the above-mentioned temperament of equal harmony, I had a monochord made in a very accurate manner, and upon it I laid down the diviſions for the thirteen notes of an octave properly tempered in the manner explained above. After a great deal of trouble in adjuſting the moveable fret, correcting the diviſions, &c. I at laſt ſucceeded ſo well as to render the diviſions exact within at leaſt the 300th part of an inch, and every part of the inſtrument was rendered ſufficiently ſteady and unalterable.

This being done, I had a large harpſichord, with a ſingle uniſon (in order to judge the better of the effect), tuned very accurately by the help of the monochord. With this inſtrument, in whatever key the performer played, the harmony was perfectly equal throughout, and the effect was the ſame as if one played in the key of E natural on a harpſichord tuned in the uſual manner.

I ſhall, therefore, conclude with ſaying, that when the harpſichord, organ, &c. is to ſerve for ſolo playing, and for a particular ſort of muſic, it is proper to tune in the uſual manner, *viz.* ſo as to give the greateſt effect to thoſe concords which occur more frequently in that ſort of muſic; but that when the inſtrument is to ſerve for accompanying other inſtruments or human voices, and eſpecially when modulations and tranſpoſitions are to be practiſed, then it muſt be tuned according to the temperament of equal harmony, which has been explained in the preceding pages.